

Radiative signature of magnetic fields in internal shocks

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ABSTRACT

Common models of blazars and gamma-ray bursts assume that the plasma underlying the observed phenomenology is magnetized to some extent. Within this context, radiative signatures of dissipation of kinetic and conversion of magnetic energy in internal shocks of relativistic magnetized outflows are studied. We model internal shocks as being caused by collisions of homogeneous plasma shells. We compute the flow state after the shell interaction by solving Riemann problems at the contact surface between the colliding shells, and then compute the emission from the resulting shocks. Under the assumption of a constant flow luminosity we find that there is a clear difference between the models where both shells are weakly magnetized ($\sigma \lesssim 10^{-2}$) and those where, at least, one shell has a $\sigma \gtrsim 10^{-2}$. We obtain that the radiative efficiency is largest for models in which, regardless of the ordering, one shell is weakly and the other strongly magnetized. Substantial differences between weakly and strongly magnetized shell collisions are observed in the inverse-Compton part of the spectrum, as well as in the optical, X-ray and 1 GeV light curves. We propose a way to distinguish observationally between weakly magnetized from magnetized internal shocks by comparing the maximum frequency of the inverse-Compton and synchrotron part of the spectrum to the ratio of the inverse-Compton and synchrotron fluence. Finally, our results suggest that LBL blazars may correspond to barely magnetized flows, while HBL blazars could correspond to moderately magnetized ones. Indeed, by comparing with actual blazar observations we conclude that the magnetization of typical blazars is $\sigma \lesssim 0.01$ for the internal shock model to be valid in these sources.

Key words: BL Lacertae objects: general – Magnetohydrodynamics (MHD) – Shock waves – radiation mechanisms: non-hermal – radiative transfer – gamma rays: bursts

1 INTRODUCTION

Highly variable radiation flux has been observed in the relativistic outflows of blazars and gamma-ray bursts (GRBs). Even though the radiation energy and time scales are different for both classes of objects (γ -rays on a millisecond timescale for GRBs versus X-rays on a timescale of hours for blazars) the underlying physics responsible for the energy dissipation might be very similar. The internal shock scenario (Rees & Meszaros 1994) has been used to explain the variability of blazars (e.g., Spada et al. 2001; Mimica et al. 2004) and GRBs (e.g., Kobayashi et al. 1997; Daigne & Mochkovitch 1998; Bošnjak et al. 2009). In this scenario inhomogeneities in a relativistic outflow cause parts of the fluid to collide and produce shocks waves which dissipate energy. The shell collisions are often idealized as collisions of dense shells. In recent years one- and two-dimensional relativistic hydrodynamics (RHD, Kino et al. 2004; Mimica et al. 2004, 2005) and relativistic magneto-hydrodynamics (RMHD, Mimica et al. 2007) simulations of the shell collisions have been performed and have showed that the dynamics of shell interaction is much more complex than what

is commonly assumed when modeling shell interactions analytically (e.g., Kobayashi et al. 1997; Daigne & Mochkovitch 1998; Spada et al. 2001; Bošnjak et al. 2009). Particularly, the influence of the magnetic field (if present) has been shown to significantly alter the dynamics (Mimica et al. 2007). In spite of these efforts, we still do not know with certainty whether the flow, whose energy is being dissipated, is significantly magnetized, or whether it is only the kinetic energy which ultimately powers the emission.

In a previous work (Mimica & Aloy 2010, MA10 in the following) we have studied the dynamic efficiency, i.e. the efficiency of conversion of kinetic to thermal and/or magnetic energy in internal shocks. We found that the dynamic efficiency is actually higher if the shells are moderately magnetized ($\sigma \approx 0.1$, see the next section for the definition of σ) than if both are unmagnetized. However, we did not compute the radiative efficiency of such interactions, but instead used the dynamic efficiency as an upper bound of it. Recently Böttcher & Dermer (2010, BD10 in the following), Joshi & Böttcher (2011) and Chen et al. (2011) have presented sophisticated models for the detailed computation of the emission from internal shocks. While these models assume a simple hydrodynamic evolution, they employ a time-dependent radiative transfer scheme which involves the synchrotron and synchrotron self-

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Compton (SSC) processes as well as the contribution Comptonized external radiation (external inverse Compton - EIC), all the while taking into account the radiative losses of the emitting non-thermal particles. We have adapted the method of BD10 and use it to perform a parametric study, addressed to infer the magnetization of the flow from the light curves and spectra of internal shocks in magnetized plasma.

The organization of this paper is as follows: Section 2 briefly summarizes the model of MA10 which is used to study the shell collision dynamics, and in Sections 3 and 4 we describe the numerical method we employ to compute the non-thermal radiation. We discuss the radiative efficiency in Section 5 and present the spectra and light curves in Section 6. A global parameter study is elaborated in Section 7. We close the paper with a discussion of our results and give our conclusions (Sec. 8).

2 SHELL COLLISION DYNAMICS

As was discussed in detail in the Section 2 of MA10, our aim is to model a large number of shell collisions with varying properties. Therefore, we employ a simplified model for a single shell collision, based on the exact solution of the Riemann problem. When describing the initial states of the Riemann problem we will use subscripts L and R to denote left (faster) and right (slower) shells, respectively.

We assume a cylindrical outflow with a radius R . Mimica et al. (2004) show that the jet lateral expansion in this case is negligible. For simplicity, and being consistent with previous work in the field (e.g., BD10, Joshi & Böttcher 2011), we also ignore the shell longitudinal expansion after the shocks cross the shell (see also Section 3.4). Following the equation 9 of MA10 we define the luminosity as

$$L := \pi R^2 \rho c^3 \left[\Gamma^2 (1 + \epsilon + \chi + \sigma) - \Gamma \right] \sqrt{1 - \Gamma^{-2}}, \quad (1)$$

where c is the speed of light in vacuum, ρ is the fluid rest-mass density, ϵ is the specific internal energy, $\chi := p/(\rho c^2)$ is the initial ratio between the thermal pressure and the rest-mass energy density, and $\sigma := B^2/(4\pi\rho\Gamma^2 c^2)$ is the magnetization parameter. Here B is the strength of the large-scale magnetic field, which is perpendicular to the direction of propagation of the fluid moving with velocity v and a corresponding Lorentz factor $\Gamma := 1/\sqrt{1 - (v/c)^2}$. The specific internal energy is related to the pressure and to the density through the equation of state. We use the TM analytic approximation to the Synge equation of state (de Berredo-Peixoto et al. 2005; Mignone et al. 2005) and obtain:

$$\epsilon := \frac{3}{2} \frac{p}{\rho c^2} + \left[\frac{9}{4} \left(\frac{p}{\rho c^2} \right)^2 + 1 \right]^{1/2} - 1. \quad (2)$$

We assume that $L_L = L_R$ and $\chi_L = \chi_R$. Furthermore, as in MA10, we assume $\Gamma_L := (1 + \Delta g)\Gamma_R$. This leaves us with R , σ_L , σ_R and Δg as parameters, because all other quantities can be determined using equations (1) and (2). To these, we add an additional parameter Δr , which is the initial width of the shells in the LAB-frame. While it does not influence the solution of the Riemann problem, it provides the physical scale necessary for the calculation of the observed emission.

Once the initial states are constructed, we compute the exact solution of the Riemann problem using the solver of Romero et al. (2005). The initial discontinuity between left and right states decomposes into a contact discontinuity (CD), and a left-going and a right wave (in the frame in which the CD is at rest). Depending on

the particulars of the initial states these waves can either be shocks or rarefactions. We label the left-going wave with RS to denote a reverse shock, and with RR in case a reverse rarefaction happens. Similarly, we label the right-going wave with FS or FR to differentiate the cases in which a forward shock or a forward rarefaction occurs, respectively. We will use a subscript S to refer to the properties of the shocked fluid in general, and the subscripts FS and RS when distinguishing between the front and reverse shocked fluids. Finally, we will use the subscript 0 for properties of the initial states in general, and the subscripts L and R when we need to distinguish between left and right initial states. Because we assume that the flow luminosity is the same for both initial states, using (1) we determine the number density in the shells to be

$$n_{L,R} = \frac{L}{\pi R^2 m_p c^3 \left[\Gamma_{L,R}^2 (1 + \epsilon + \chi + \sigma_{L,R}) - \Gamma_{L,R} \right] \sqrt{1 - \Gamma_{L,R}^{-2}}}, \quad (3)$$

where $\Gamma_L = (1 + \Delta g)\Gamma_R$.

The Riemann solver provides us with the bulk velocity of the shocked fluid βc (and its Lorentz factor $\Gamma = (1 - \beta^2)^{-1/2}$), and velocities $\beta_{FS} c$ and $\beta_{RS} c$ of the FS and RS, respectively (provided they exist). The velocity of the initial (unshocked) states in the CD rest frame is

$$\beta'_0 = \frac{\beta_0 - \beta}{1 - \beta\beta_0}. \quad (4)$$

The shock velocities in the frame of the CD can be computed as

$$\beta'_S = \frac{\beta_S - \beta}{1 - \beta\beta_S}, \quad (5)$$

where prime denotes quantities in the CD rest frame. In this frame the shock crosses the shell at a time

$$t'_{\text{cross},S} = \frac{\Delta r'_0}{c |\beta'_S|}, \quad (6)$$

where $\Delta r'_0$ is the shell width in the CD frame,

$$\Delta r'_0 = \Gamma \Delta r \frac{\beta - \beta_S}{\beta_0 - \beta_S}. \quad (7)$$

3 NON-THERMAL PARTICLES

In this section we show the properties of non-thermal particles and their emission. We first discuss the model for the magnetic field and non-thermal particles, and then outline the method used to compute their emission.

3.1 Magnetic field

As in Mimica & Aloy (2010) and BD10, we assume that there exists a stochastic magnetic field, which is created *in situ* by the shocks arising in the collision of the shells. We label this field by $B_{S,\text{st}}$, and by definition its strength is a fraction ϵ_B of the internal energy density of the shocked shell u_S (obtained, in our case, by the exact Riemann solver):

$$B_{S,\text{st}} = \sqrt{8\pi\epsilon_B u_S}. \quad (8)$$

Since we allow for arbitrarily magnetized shells, there is also an ordered (macroscopic) magnetic field component $B_{S,\text{mac}}$, which is a direct output of the exact Riemann solver. The total magnetic field is then $B_S := \sqrt{B_{S,\text{st}}^2 + B_{S,\text{mac}}^2}$.

3.2 Injection spectrum of non-thermal particles

We assume that a fraction of electrons in the unshocked shell is accelerated to high energies at the shock front. Following Section 3 of BD10, we assume that a fraction ϵ_e of the dissipated kinetic energy is used to accelerate electrons. We assume that some particle acceleration mechanism operates at shocks, such that a fraction of the electrons in the unshocked shell is accelerated to high energies in the vicinity of the shock front. As it is commonly done (Bykov & Meszaros 1996; Daigne & Mochkovitch 1998; Mimica et al. 2004, BD10), we assume that a fraction ϵ_e of the dissipated kinetic energy is used to accelerate electrons. The width of the acceleration zone $\Delta r'_{\text{acc}}$ is parametrized as a multiple Δ_{acc} of the proton Larmor radius in the shocked fluid. Therefore, the acceleration at a given point in the shocked fluid lasts for a time $\Delta t'_{\text{acc}} = \Delta r'_{\text{acc}}/(\beta'_S c)$. We have

$$\Delta r'_{\text{acc}} = \Delta_{\text{acc}} \frac{\Gamma'_0 m_p c^2}{e B_S}, \quad (9)$$

From this expression, we compute the volume where acceleration takes place as $V'_{\text{acc}} = \pi R^2 \Delta r'_{\text{acc}}$. The energy injection rate into the acceleration region is

$$\frac{dE'_{\text{inj},0}}{dt'} = \pi R^2 \epsilon_e u_S \frac{\Delta r'_{\text{acc}}}{\Delta t'_{\text{acc}}}, \quad (10)$$

and we assume that the energy spectrum of the injected relativistic particles is a power-law in the electron Lorentz factor γ ,

$$\frac{dn'_{\text{inj}}}{d\gamma} = Q_0 \gamma^{-q} H(\gamma; \gamma_1, \gamma_2), \quad (11)$$

where n'_{inj} is the number density of the injected electrons, Q_0 is a normalization factor and γ_1 and γ_2 are the lower and upper injection cutoffs (computed below), all measured in the shocked fluid rest frame. The step function is defined as usual by $H(x; a, b) = 1$ if $a \leq x \leq b$ and 0 otherwise.

A cautionary note should be added here regarding the fact that we choose that the spectral energy distribution of the injected particles is a pure power-law even in the high- σ regime. Both theoretical arguments (e.g., Kirk & Heavens 1989) and recent particle-in-cell (PIC) simulations (e.g., Sironi & Spitkovsky 2009) have shown that particle acceleration is not very efficient in the presence of a strong magnetic field parallel to the shock front. The modifications to the particle injection spectrum might involve the presence of the thermal population. Recently, a calculation by Giannios & Spitkovsky (2009) shows how the spectrum of the GRB prompt emission might look in such a case: a bump at the spectral maximum and a lower contribution at ultra-high energies. However, current PIC simulations have not been run for sufficiently long time to achieve a stable situation. Thus, the fraction of the energy which goes into thermal electrons (parameter δ of Giannios & Spitkovsky 2009) is still to be determined. In this particular study we set this fraction to zero and thus avoid introducing another free parameter in our models.

Integrating the distribution Eq. 11 in Lorentz factor and equating the result to Eq. 10 divided by V'_{acc} (in order to obtain the energy density injection rate into the acceleration region) we can compute the normalization factor for the electron injection,

$$Q_0 = \frac{dE'_{\text{inj},0}/dt'}{V'_{\text{acc}} m_e c^2} \times \begin{cases} \frac{q-2}{\gamma_1^{2-q} - \gamma_2^{2-q}} & \text{if } q \neq 2 \\ 1/\ln\left(\frac{\gamma_2}{\gamma_1}\right) & \text{if } q = 2 \end{cases}. \quad (12)$$

3.3 Particle injection cut-offs

As was done in Mimica et al. (2010), we assume that the upper cutoff for the electron injection is obtained by assuming that the acceleration time scale is proportional to the gyration time scale. Then the maximum Lorentz factor is obtained by equating this time scale to the cooling time scale,

$$\gamma_2 = \left(\frac{3m_e^2 c^4}{4\pi a_{\text{acc}} e^3 B_S} \right)^{1/2}. \quad (13)$$

where $a_{\text{acc}} \geq 1$ is the acceleration efficiency parameter (BD10). The lower cut-off is obtained by assuming, in complete analogy to Eq. 10, that the number of accelerated electrons is related to the number of electrons passing through the shock front,

$$\frac{dN'_{\text{inj},0}}{dt'} = \zeta_e \pi R^2 n_0 \Gamma'_0 \beta'_S c, \quad (14)$$

where ζ_e is the fraction of electrons accelerated into the power-law distribution. From Eqs. 14, 10 and 11 we get

$$\frac{\int_{\gamma_1}^{\gamma_2} d\gamma \gamma^{1-q}}{\int_{\gamma_1}^{\gamma_2} d\gamma \gamma^{-q}} = \frac{\epsilon_e}{\zeta_e} \frac{u_S}{\Gamma'_0 n_0 m_e c^2}. \quad (15)$$

Since we are dealing with potentially highly magnetized fluids, the condition $\gamma_2 \gg \gamma_1$ cannot be assumed (see Eq. 13), and therefore we cannot use the equation such as Eq. 13 of BD10. Therefore we compute γ_1 from Eq. 15 numerically using an iterative procedure.

3.4 Evolution of the particle distribution

In this work we assume that particles cool via synchrotron and external-Compton processes. We ignore the adiabatic cooling in this work since we are primarily interested in collisions of magnetized shells, where the electrons are fast-cooling. The consequence of not accounting for the adiabatic losses of the particle distribution is that our method overestimates the emission after the shocks cross the shells. Nevertheless, most of the features that make substantive differences between the dynamics triggered by magnetized and non-magnetized shells happen in the early light curve and, thus, neglecting the expansion of the shells plasma does not change the qualitative conclusions of this paper.

The radiative losses for an electron with a Lorentz factor γ can be written as

$$\dot{\gamma} = -\frac{4}{3} c \sigma_T \frac{u'_B + u'_{\text{ext}}}{m_e c^2} \gamma^2, \quad (16)$$

where $u'_B = B_S^2/8\pi\Gamma^2$ and u'_{ext} are the energy density of the magnetic field and the external radiation field (see Sec. 4.2) in the shocked fluid frame, respectively. Once the energy losses have been specified, we use the semi-analytic solver of Mimica et al. (2004) to compute the particle distribution at any time after the start of the injection at shock. More precisely, we use the solution for the continuous injection and particle cooling (Eq. 19 of Mimica et al. 2004) for the time $\Delta t'_{\text{acc}}$ since the beginning of the shock acceleration. After that time the shock acceleration ends and we approximate the resulting particle distribution by a piecewise power-law function. Then we employ Eq. 17 of Mimica et al. (2004) on each of the power-law segments to compute the subsequent evolution.

4 NON-THERMAL RADIATION

We assume that the observer's line of sight makes an angle θ with the jet axis, which is also the direction of propagation of the fluid.

We denote by x and t the position and time in the observer frame, and by x' and t' the location and time in the CD frame. We assume that the CD is located at $x' = 0$ for all t' . Let $\mu \equiv \cos \theta$, and define the time at which an observer sees the radiation emitted from x at time t as

$$T = t - x\mu/c. \quad (17)$$

Then the time as a function of the time of observation and position can be written as

$$t' = \mathcal{D}(T/(1+z) + \Gamma(\mu - \beta)x'/c), \quad (18)$$

where $\mathcal{D} := [\Gamma(1 - \beta\mu)]^{-1}$ is the Doppler factor and z is the redshift. Lorentz transformations have been applied to Eq. 17 to obtain Eq. 18.

An important quantity is the time elapsed after the shock has passed a given x' . From such value one can calculate the *age* of the electron distribution function at that position, which turns to be the time since the shock acceleration has begun. Thus, the age can be defined as

$$t'_a := t' - \frac{x'}{\beta'_s c}. \quad (19)$$

A more useful expression involves T . Using Eqs. 19 and 18 we get, for the FS

$$t'_{a,FS} = \mathcal{D} \left[\frac{T}{1+z} - \frac{x'}{c\Gamma} \frac{1 - \beta_{FS}\mu}{\beta_{FS} - \beta} \right], \quad (20)$$

and for the RS

$$t'_{a,RS} = \mathcal{D} \left[\frac{T}{1+z} - \frac{x'}{c\Gamma} \frac{1 - \beta_{RS}\mu}{\beta_{RS} - \beta} \right]. \quad (21)$$

We note that Eq. 20 has to be used when $x' \geq 0$, while Eq. 21 is valid when $x' < 0$. If $t'_a \leq 0$, then the shock has not crossed that position yet and, consequently, that place does not contribute to the emission yet.

The observed luminosity in the CD rest frame is

$$\nu' L'_{\nu'}(T) = \pi R^2 \int_{x'_{\min}(T)}^{x'_{\max}(T)} dx' \nu' j'_{\nu'}[t'_a(T, x')], \quad (22)$$

where the lower and upper limits depend on (1) whether the shock exists, and (2) whether it has crossed the shell¹. If the RS does not exist, then $x'_{\min} = 0$; otherwise it is

$$x'_{\min}(T) = \max \left(\frac{\Gamma c T}{1+z} \frac{\beta_{RS} - \beta}{1 - \beta_{RS}\mu}, -\Delta r'_L \right), \quad (23)$$

where $\Delta r'_L$ is the faster shell width in the CD frame. Analogously, for $x'_{\max} = 0$ the FS is non-existent; otherwise it is

$$x'_{\max}(T) = \min \left(\frac{\Gamma c T}{1+z} \frac{\beta_{FS} - \beta}{1 - \beta_{FS}\mu}, \Delta r'_R \right), \quad (24)$$

$\Delta r'_R$ being the slower shell width in the CD frame. We point out that, to perform the integral in Eq. 22, $j'_{\nu'}(t'_a)$ should be computed for the particle distribution evolved using values for the FS if $x' > 0$,

¹ Since the assumed geometry in this model is cylindrical, the effects of the high-latitude emission are ignored. This means that all the peaks and breaks in the light curves are sharper than would be in case a conical geometry was assumed. Another consequence of the assumed geometry is that we overestimate the rate at which the light curve declines after shocks cross the shells (see Section 6.2).

and RS if $x' < 0$ (i.e. in Eqs. 12, 13, 15 and 8 the values for the corresponding shocked fluid should be used).

Considering that $\nu' = \nu(1+z)/\mathcal{D}$, and using Eq. 22, we can compute the flux in the observer frame obtaining (BD10, Dermer 2008)

$$\nu F_{\nu}(T) = \frac{\mathcal{D}^4 \pi R^2}{d_L^2} \int_{x'_{\min}(T)}^{x'_{\max}(T)} dx' \nu' j'_{\nu'}[t'_a(T, x')], \quad (25)$$

where d_L is the luminosity distance. We perform the integration in Eq. 25 numerically.

The total emissivity is assumed to be the result of combining three emission processes: (1) synchrotron radiation, (2) inverse Compton with an external radiation field (EIC), and the synchrotron self-Compton (SSC) up-scattering. These emission processes are considered in more detail in the next sections.

4.1 Synchrotron emission

We compute the synchrotron emission for each power-law segment of the electron distribution (see Sec. 3.4) separately. In order to speed up the calculation we use the interpolation method described in Mimica et al. (2009, section 4) and, in more detail, in Mimica (2004, sections 2.1.3 and 4.3.1).

4.2 EIC emission

Following BD10, we assume that the external radiation field is monochromatic and isotropic in the AGN frame. We denote the frequency and the radiation field energy density in this frame by ν_{ext} and u_{ext} , respectively. Transforming into the shocked fluid frame we get

$$\begin{aligned} \nu'_{\text{ext}} &= \Gamma \nu_{\text{ext}} \\ u'_{\text{ext}} &= \Gamma^2 u_{\text{ext}} \end{aligned} \quad (26)$$

Analogously to the computation of the synchrotron emission (Sec. 4.1), we compute the EIC emissivity for each power-law segment separately. We use Eq. 2.94 of Mimica (2004), but replacing $I(\nu_0)/\nu_0$ by $cu'_{\text{ext}}/\nu'_{\text{ext}}$ and with an additional cut-off (approximating the Klein-Nishina decline of the Compton cross-section) such that the emissivity is zero for $h\nu \geq m_e^2 c^4 / (h\nu'_{\text{ext}})$ (see also, Aloy & Mimica 2008). Values of ν_{ext} and u_{ext} used in this work can be found in Table 1.

4.3 SSC emission

Analogously to Sec. 4.2, we use the Eq. 2.94 of Mimica (2004). However, in the case of SSC the incoming intensity of the synchrotron radiation depends on x' and T . For a point on the shell axis the (angle averaged) intensity at frequency ν_0 can be written as

$$I_{0,\nu_0}(T, x') = \frac{1}{2} \int_0^\pi d\theta' \int_0^{L(\theta')} ds j'_{\nu_0,\text{syn}}(t'(T) - s/c, x' + s \cos \theta'), \quad (27)$$

where $L(\theta')$ is the length of the segment in direction θ' from which synchrotron emission has had time to arrive to x' at a time t' , and $t'(T)$ is computed using Eq. 18. The synchrotron emissivity $j'_{\nu_0,\text{syn}}$ can be rewritten in terms of T using Eqs. 19, and 20 (or 21),

$$\begin{aligned} j'_{\nu_0,\text{syn}} \left(t'(T) - \frac{s}{c}, x' + s \cos \theta' \right) = \\ j'_{\nu_0,\text{syn}} \left[t'_a(T, x') - \frac{s}{c} \left(1 + \cos \theta' \frac{1 - \beta\beta_s}{\beta_s - \beta} \right) \right], \end{aligned} \quad (28)$$

Parameter	value
Γ_R	10
Δg	1
σ_L	$[10^{-6}, 10^1]$
σ_R	$[10^{-6}, 10^1]$
ϵ_B	10^{-3}
ϵ_e	10^{-1}
ζ_e	10^{-2}
Δ_{acc}	10
a_{acc}	10^6
R	3×10^{16} cm
Δr	6×10^{13} cm
q	2.6
L	5×10^{48} erg s $^{-1}$
u_{ext}	5×10^{-4} erg cm $^{-3}$
ν_{ext}	10^{14} Hz
z	0.5
θ	5°

Table 1. Blazar model parameters used in this work. Note that σ_L and σ_R can vary continuously in the indicated range.

From Eq. 28 we can see that $L(\theta')$ can be computed by requiring that the following condition be satisfied for each θ' ,

$$t'_a(T, x') - \frac{s}{c} \left(1 + \cos \theta' \frac{1 - \beta \beta_s}{\beta_s - \beta} \right) > 0.$$

If this condition is not satisfied, it means that the shock has not passed the point $x' + s \cos \theta'$ at time $t'(T) - s/c$ yet, i.e. there is no synchrotron emission from that point to contribute to the incoming intensity. In addition, we also require that $L(\theta') \leq R$. Finally, it should not be forgotten that when $x' + s \cos \theta' > 0$ the emissivity of fluid shocked by the FS should be used, and the one corresponding to the shocked fluid by the RS otherwise. Also, if either of the shocks is not present, there is no contribution from the corresponding region.

In practice the numerical cost of a direct evaluation of double integral in Eq. 27 is prohibitive if we take into account that this intensity has to be evaluated for each x' in Eq. 25. To overcome this problem we approximate Eq. 27 by discretizing the angular integral in a non-uniform θ' -intervals. The choice of non-uniform intervals is motivated by the fact that most of the contribution of the incoming radiation comes from angles close to $\mu = -\beta_s$, so that we concentrate most of the bins close to that angle. Numerical testing shows that using 13 bins provides an acceptable tradeoff between the accuracy and the computational requirements.

5 RADIATIVE EFFICIENCY

In this section we compare the radiative efficiency of the internal shocks with their corresponding dynamic efficiency. We use the kinematic parameters from MA10 in the blazar regime, while the parameters used to compute emission are guided by the values from BM10 (see Table 1 for the complete list). All parameters of our models are fixed except for σ_L and σ_R , which can vary in the range indicated by the Table 1. In the rest of the paper we distinguish models by the value of the magnetization of each shell, e.g., a model with $\sigma_L = 0.1$ and $\sigma_R = 1$ is denoted by the pair (0.1, 1).

As can be seen from Eq. 10, in our model only the thermal energy can be injected into the non-thermal particle population.

We point out that, alternatively or simultaneously, magnetic dissipation can provide a source for the emission in internal shocks (e.g., Giannios et al. 2009; Nalewajko et al. 2011, and references therein), which we are not considering here. Thus, the radiative efficiency we compute in this paper is only a lower bound to the actual radiative efficiency of the binary collision of relativistic magnetized shells. As is shown in the Appendix A, we can use the definition of the dynamic efficiency inspired by the recent work of Narayan et al. (2011). Its advantage is the Lorentz-invariance, which enables us to compare it to the radiative efficiency of our model.

Following MA10, but using the definitions for the different energy components \hat{E}_K , \hat{E}_T and \hat{E}_M of the Appendix (Eq. A2), we denote by \hat{E}_0 the total energy in the unshocked shells,

$$\begin{aligned} \hat{E}_0 &:= \hat{E}_K(\Gamma_R(1 + \Delta g), n_L m_p, 1) + \hat{E}_T(\Gamma_R(1 + \Delta g), n_L m_p, \chi n_L m_p c^2, 1) \\ &+ \hat{E}_M(\Gamma_R(1 + \Delta g), n_L m_p, \sigma_L, 1) + \hat{E}_K(\Gamma_R, n_R m_p, 1) \\ &+ \hat{E}_T(\Gamma_R, n_R m_p, \chi n_R m_p c^2, 1) + \hat{E}_M(\Gamma_R, n_R m_p, \sigma_R, 1). \end{aligned} \quad (29)$$

where χ is the pressure-to-density ratio of the cold initial shells and is set to 10^{-4} . We also define the width of the shocked shells in terms of their initial width,

$$\zeta_w := \frac{|\beta - \beta_s|}{\beta_0 - \beta_s} \quad (30)$$

so that $0 \leq \zeta_w \leq 1$.² The dynamic thermal efficiency for the faster shell is defined as

$$\epsilon_{T,L} := \frac{1}{\hat{E}_0} \times \left[\hat{E}_T(\Gamma_{S,L}, n_{S,L} m_p, p_{S,L}, \zeta_{w,L}) - \hat{E}_T(\Gamma_R(1 + \Delta g), n_L m_p, \chi n_L m_p c^2, 1) \right] \quad (31)$$

and analogous definitions can be written for $\epsilon_{M,L}$, $\epsilon_{T,R}$ and $\epsilon_{M,R}$ (see Eqs. 13, 14, 16 and 17 of MA10). The total (Lorentz invariant) dynamic thermal and magnetic efficiency is

$$\epsilon_T = \epsilon_{T,L} + \epsilon_{T,R}, \quad (32)$$

$$\epsilon_M = \epsilon_{M,L} + \epsilon_{M,R}. \quad (33)$$

From these equations it can be seen that the radiative efficiency can be at most $\epsilon_e(\epsilon_T + \epsilon_M)$. More formally, we can write the radiative efficiency as (neglecting adiabatic cooling)

$$\epsilon_{\text{rad}} := \epsilon_e f_{\text{rad}}(\epsilon_T + \epsilon_M), \quad (34)$$

where $f_{\text{rad}} := \epsilon_T / (\epsilon_T + \epsilon_M)$. It should be noted that Eq. 34 refers to the “bolometric” emission, i.e. it includes all frequencies for the whole duration of the shell interaction. Since Earth-based observations have a limited spectral and temporal coverage, the Eq. 34 is only an upper limit for radiative efficiencies inferred from actual observations. Figure 1 shows that the radiative efficiency is not a one-to-one map of the total dynamic efficiency. In particular, we note that f_{rad} drops to under 10% in the region ($\sigma_L > 10, \sigma_R > 10$). Furthermore, there is a region of maximal dynamic efficiency for $\sigma_R \approx 0.2$ and $\sigma_L > 1$, where the radiative efficiency from internal shocks, which can only tap the thermal energy in the regions downstream shocks, is not maximum. Nevertheless, for small-to-moderate values of the magnetizations of both shells (lower right quadrant of Fig. 1), the radiative efficiency is a good proxy of the total dynamical efficiency.

² Note that comparing Eqs. 7 and 30 we see that $\Delta r'_0 = \Gamma \zeta_w \Delta r$, which is just a Lorentz transformation of the shocked shell width from lab to CD frame.

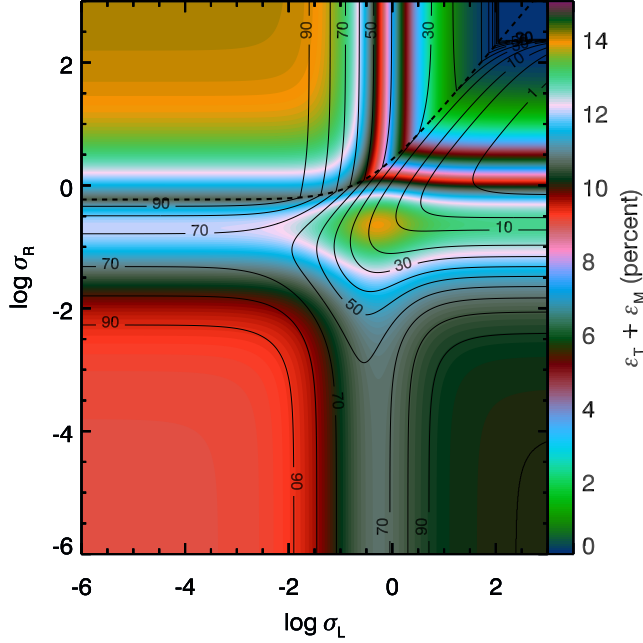


Figure 1. Contours: f_{rad} (see Eq. 34) for different values of (σ_L, σ_R) . The contours indicate the following values of f_{rad} (per cent): 1, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, and 100. In the region of the parameter space above the dashed line there is no FS, while the RS is always present for the considered parametrization (see also Fig. 1 of MA10). Filled contours: total dynamic efficiency $\epsilon_T + \epsilon_M$ in per cent.

6 SPECTRA AND LIGHT CURVES OF MAGNETIZED INTERNAL SHOCKS IN BLAZARS

Our aim is to produce synthetic spectra and light curves from our numerical models of the interaction of two relativistic, magnetized shells. With this purpose, we chose three models from our parameter space, which are representative of different conditions that can be encountered in blazar jets. The first model corresponds to a regime of very low magnetization of both shells $(\sigma_L, \sigma_R) = (10^{-6}, 10^{-6})$. The second and third models correspond to intermediate $(10^{-2}, 10^{-2})$ and moderate/high shell magnetizations $(1, 0.1)$.

6.1 Average spectra

The spectrum of the weakly magnetized model $(10^{-6}, 10^{-6})$, (Fig. 2; full lines) reproduces the typical double-peaked spectrum of blazars. The synchrotron emission (Fig. 2; solid red line) peaks at 4.6×10^{13} Hz, while the inverse-Compton (IC) emission, dominated by the SSC component, peaks at 6.7×10^{21} Hz (Fig. 2; solid blue line). In this case, the IC spectral component is clearly dominating the overall spectrum. At intermediate magnetizations (Fig. 2; dashed lines) the synchrotron emission peaks at higher frequencies than in the previous case, namely, at 7.5×10^{14} Hz, while the IC emission peaks at 6.0×10^{20} Hz, as one would expect, since a larger magnetic field increases the synchrotron peak frequency (e.g., Mimica 2004). For these shell magnetizations, the SSC also dominates the high energy emission, but now both the SSC and EIC components are significantly weaker than in the model $(10^{-6}, 10^{-6})$. Interestingly, moderate to high magnetizations (Fig. 2; dot-dot-dashed lines) reduce the peak frequency (1.9×10^{14} Hz) and substantially flatten the synchrotron spectral component w.r.t. the inter-

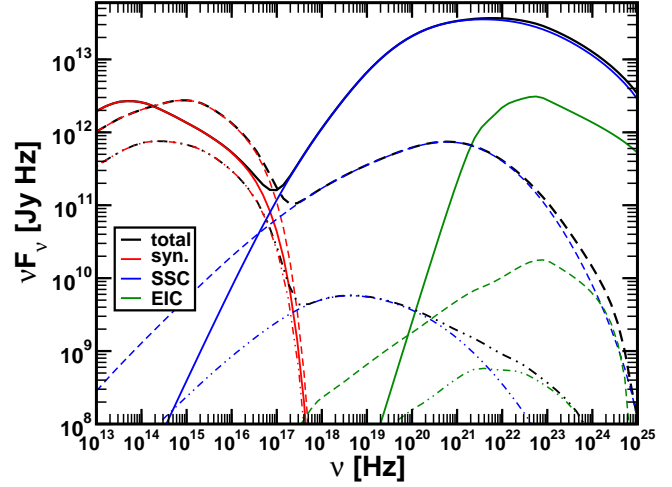


Figure 2. Averaged spectra (time integration interval: 0 - 100 ks) for models $(10^{-6}, 10^{-6})$, $(10^{-2}, 10^{-2})$ and $(1, 0.1)$ (full, dashed, and dot-dot-dashed lines, respectively). Black colored lines show the total spectrum, while the red, blue and green lines show the contribution due to synchrotron, SSC and EIC emission, respectively.

mediate magnetization case. Furthermore, IC spectral components are notably weaker than the synchrotron one for large magnetization, being the IC spectral peak located at 4.2×10^{18} Hz.

As can be seen from Fig. 2, the synchrotron emission from all three models is of comparable intensity, while the IC emission is much weaker in the strongly magnetized model $(1, 0.1)$. The reason for such a large difference in the high-energy emission between the magnetized and the non-magnetized models lies in the lower number density of emitting electrons (Eq. 3) and in the higher magnetic field of the magnetized model. The magnetized model has a much lower number density due to its relatively high σ , in both FS and RS emitting regions, which means that there are less scatterers for the SSC and EIC processes. A high magnetic field also means a reduction of the upper injection cut-off (Eq. 13), which in turn means that the seed synchrotron photons in the SSC process are being up-scattered to lower frequencies, explaining the small contribution of the SSC component to the spectrum. The EIC component's upper cut-offs are determined by the Klein-Nishina decline (see Sec. 4.2) and not by the upper injection cut-off, which explains why the EIC spectral peaks of the models are in a similar frequency range.

Figure 3 shows the contributions from the FS and RS (red and blue lines, respectively) to the total spectrum (black lines). Except close to the local minima between the two spectral peaks, the RS contribution is dominant in the average spectra of the models with low and intermediate magnetizations, $(10^{-6}, 10^{-6})$ and $(10^{-2}, 10^{-2})$ respectively. In the vicinity of the aforementioned minima (located in the X-rays range), the FS contribution tends to broaden the width of the minima and to soften the spectral slope. At the moderate to high magnetizations of the model $(1, 0.1)$ the FS is dominant except in the range $10^{15} - 10^{16}$ Hz, where the FS and the RS have comparable contributions. The reason is that the faster shell, through which the RS propagates, is substantially more magnetized than the slower shell, so that the RS has less particles to accelerate than the FS.

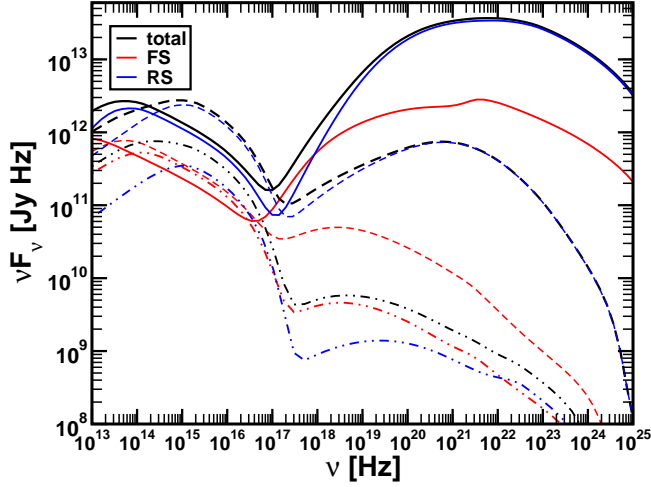


Figure 3. Same as Fig. 2, but distinguishing the contributions of the FS (red lines) and of the RS (blue lines) to the total spectrum (black lines).

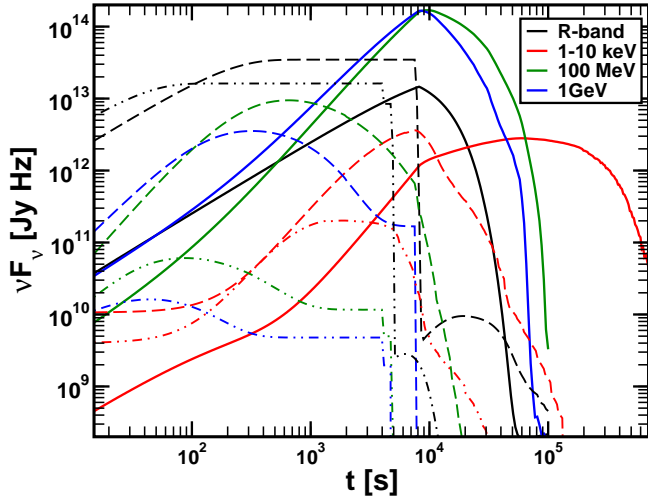


Figure 4. Multi-wavelength light curves for the models $(10^{-6}, 10^{-6})$, $(10^{-2}, 10^{-2})$ and $(1, 0.1)$, shown with full, dashed and dot-dot-dashed lines, respectively. The R-band (5×10^{14} Hz), X-ray band ($1 - 10$ keV), as well as 0.1 GeV and 1 GeV light curves are shown in black, red, green and blue, respectively.

6.2 Light curves

The multi-wavelength light curves of the models presented in the previous section are displayed in Fig. 4. We have picked up several characteristic bands to analyse the data (R-band, X-ray, 0.1 GeV and 1 GeV).

Comparing the R-band light curves of the three models we can see that models $(10^{-2}, 10^{-2})$ and $(1, 0.1)$ exhibit properties of the fast-cooling electrons emitting synchrotron radiation, while the model $(10^{-6}, 10^{-6})$ shows the opposite, slow-cooling behavior. In the latter case the maximum of the R-band light curve is reached when the shocks cross the shells, and afterward the emission decays as the particles cool down (no new particles are accelerated after both shocks cross the shell). In the case of the model $(1, 0.1)$ one can clearly notice two sudden drops in emission around 4 ks, which correspond to the moments when first the RS, and later, the FS cross their respective shells. The almost vertical drops in emission are indicative of a very efficient electron synchrotron-cooling

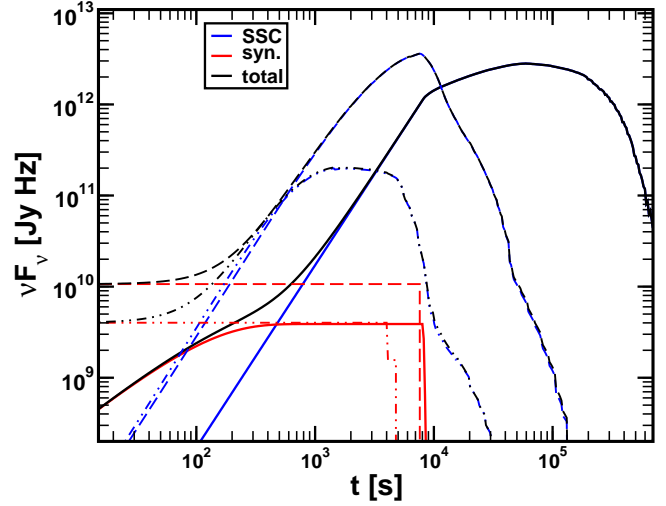


Figure 5. X-ray light curves for the models $(10^{-6}, 10^{-6})$, $(10^{-2}, 10^{-2})$ and $(1, 0.1)$, shown with full, dashed and dot-dashed lines, respectively. The total light curve is shown in black, while the synchrotron and SSC contributions are shown in red and blue, respectively.

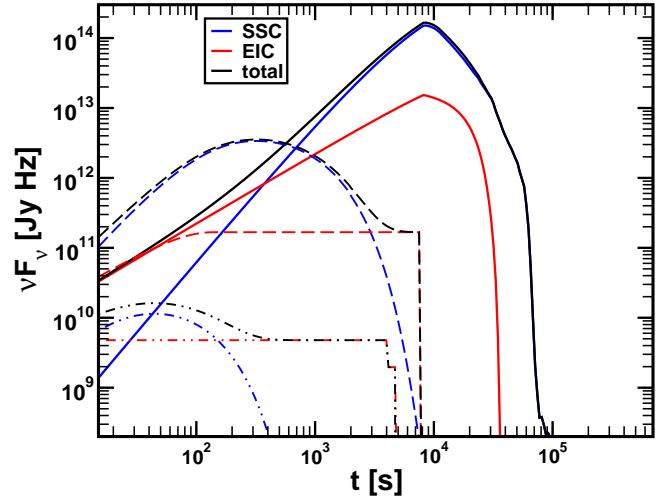


Figure 6. 1 GeV light curves for the models $(10^{-6}, 10^{-6})$, $(10^{-2}, 10^{-2})$ and $(1, 0.1)$, shown with full, dashed and dot-dot-dashed lines, respectively. The total light curve is shown in black, while the EIC and SSC contributions are shown in red and blue, respectively.

³ At intermediate magnetizations $(10^{-2}, 10^{-2})$ the first sharp drop is observable as well, though here there is a weak late-time optical emission between 10^4 and 10^5 seconds due to the SSC process.

The emission in the X-ray band is a bit more involved, and to perform a proper analysis we show in Fig. 5 both, the total light curve (black lines), and the individual contributions to it of the synchrotron and SSC processes (red and blue lines, respectively)⁴. Except at very early times, the emission is dominated by the SSC process in all cases. The synchrotron emission in this band happens

³ Note that, since the high-latitude emission is ignored due to cylindrical geometry, the drops are too sharp and would be smoother were a conical jet geometry assumed.

⁴ We do not show the EIC light curve because its contribution is negligible at these frequencies.

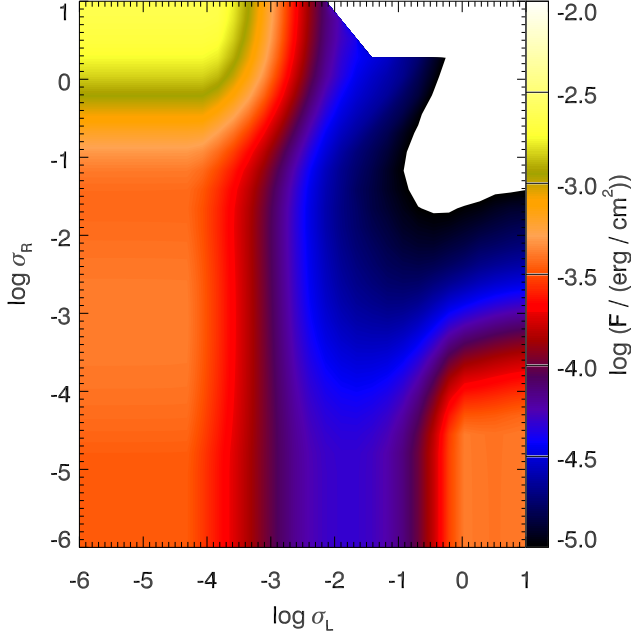


Figure 7. Contours of the logarithm of the time (0 – 100 ks) and frequency ($10^{12} - 10^{25}$ Hz) integrated flux (i.e., fluence) as a function of the shell magnetization σ_L and σ_R . Note that, different from Fig. 1, the region of ultra-high magnetizations ($\sigma_{L,R} > 10$) is not included in this figure. The reason being that the computation of the integrated flux with such extreme magnetic fields requires a discretization of the two-dimensional integral in Eq. 27 in a very large number of intervals, making the calculation numerically impractical. Nevertheless, the trends at such high magnetizations can be easily extrapolated from the values displayed in the figure.

in an efficient fast-cooling regime, which can be inferred from the fast drop of the synchrotron components between 4 and 9 ks. The fact that increasing magnetic fields make that particles cool faster, explains that the non-magnetized model peaks much later (≈ 60 ks) in this band than the other two (more magnetized) models.

At energies of 1 GeV, there is only emission from IC processes (Fig. 6). The model with the smaller magnetization displays a clearly dominant EIC emission at early times, while in the other two models EIC dominates the later times. In the models (10^{-2} , 10^{-2}) and (1, 0.1) EIC, similar to the synchrotron emission in the X-ray band, sinks very quickly before 8 ks, indicating that the electrons are in a fast-cooling regime. In the latter models, because of the delays associated to the physical length of the emitting region, the SSC contribution peaks very early and decays exponentially before the sharp drop of the EIC emission (this is particularly the case of the most magnetized model, in which the SSC component does not significantly contribute to the light curve after ≈ 400 s). In contrast, the EIC emission of the model (10^{-6} , 10^{-6}) shows a much more prominent peak and a shallower decay from the maximum (at ≈ 9 ks), both features being characteristics from electrons in a slow-cooling regime.

7 GLOBAL PARAMETER STUDY

In the following we present the results of the global parameter study of the dependence of the emitted radiation on the shell magnetiza-

tion⁵. Figure 7 shows the fluence as a function of σ_L and σ_R . We can see that, as expected, the fluence roughly follows f_R (see Sect. 5). The region with most luminous internal shocks (upper left corner of Fig. 7) happens for a moderately-to-strongly magnetized slow shell and a weakly magnetized fast shell, whereby the FS does not exist. The emission weakens as the magnetization of the fast shell increases, with the exception of the region where the fast shell is strongly magnetized but the slow shell is weakly magnetized (lower right corner of Fig. 7). We conclude that, as was indicated in the Section 4.4 of MA10, a large difference in the magnetization of the shells yields stronger dissipation and more luminous internal shock(s) than when both shells are weakly magnetized.

Figure 8 shows the spectral maxima of the synchrotron, $\nu_{\max,\text{syn}}$, and the inverse Compton emission, $\nu_{\max,\text{IC}}$ (left and right panels, respectively). From Fig. 2, one could anticipate a trend we confirm here, with the parametric space coverage, namely, that the IC emission is more sensitive to changes in the magnetization than the synchrotron emission. This statement reflects itself in Fig. 8 through the fact that the range of variation of $\nu_{\max,\text{IC}}$ is larger than that of $\nu_{\max,\text{syn}}$. Thus, the IC spectral peak becomes a better proxy of the magnetization of the shells than the synchrotron peak. Except at small shell magnetizations, the IC emission happens in a fast-cooling regime, and the dependence of $\nu_{\max,\text{IC}}$ with the magnetization is similar to that of f_{rad} . Complementary, at small shell magnetizations, the map of $\nu_{\max,\text{syn}}$, resembles very much to that of f_{rad} (compare the lower half of Figs. 1 and 8-left).

The left panel of the figure 9 shows the ratio of the IC to synchrotron fluence. The trend is quite similar to that of the integrated flux shown in Fig. 7. When both shells are strongly magnetized ($\sigma \gtrsim 0.1$) the IC emission drops significantly. In the region $\sigma_L \lesssim 10^{-3}$ the ratio is between a unity and ≈ 60 , with a similar behavior in the region ($\sigma_L \gtrsim 0.1, \sigma_R \lesssim 0.1$). The region of low radiative efficiency around $\sigma_L \approx 0.01$ appears as a dark vertical band in the plot ($\sigma_R \lesssim 10^{-4}$, where both synchrotron and IC processes provide a similar fluence). The right panel of Fig. 9 shows the ratio of the frequencies of the IC and synchrotron spectral maxima shown in Fig. 8. From an observational point of view, the fluence might be much more robust and significant than the peak IC and synchrotron frequencies, which can be difficult to measure. However, for the ratio $\nu_{\text{IC}}/\nu_{\text{syn}}$, the lower right and upper left corners of the plot display noticeably different values. Thus, one can use the frequency ratio together with the fluence ratio in order to break the degeneracy in the fluence ratio when one of the shell is very magnetized and the other is not magnetized (see Sec. 8.2 for further discussion of this point).

8 DISCUSSION AND SUMMARY

We have extended the study of the dynamic efficiency performed in MA10 by computing the multi-wavelength, time-dependent emission from internal shocks. In this section we discuss and summarize our findings.

⁵ All two-dimensional plots in this section have been produced using a logarithmically spaced grid of 30×30 in the $\sigma_L \times \sigma_R$ parameter space. For each of the points we computed light curves on 96 logarithmically spaced frequencies (between 10^{12} Hz and 3×10^{25} Hz) for 120 logarithmically spaced points in time (between 2 and 10^5 seconds). A finer coverage of the $\sigma_L \times \sigma_R$ parameter space was not practical due to the prohibitively high memory and computational time requirements on the available machines.

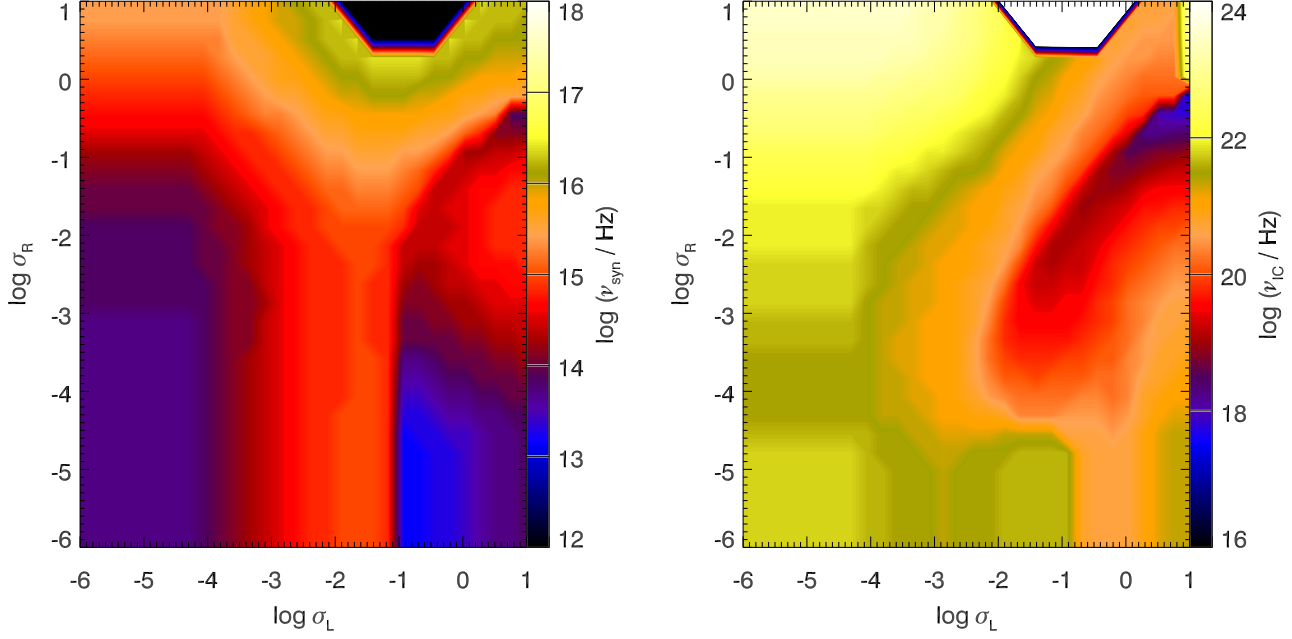


Figure 8. Contours of the frequency of the spectral maxima of the synchrotron (left panel) and of the inverse Compton (right panel) emission as a function of σ_L and σ_R .

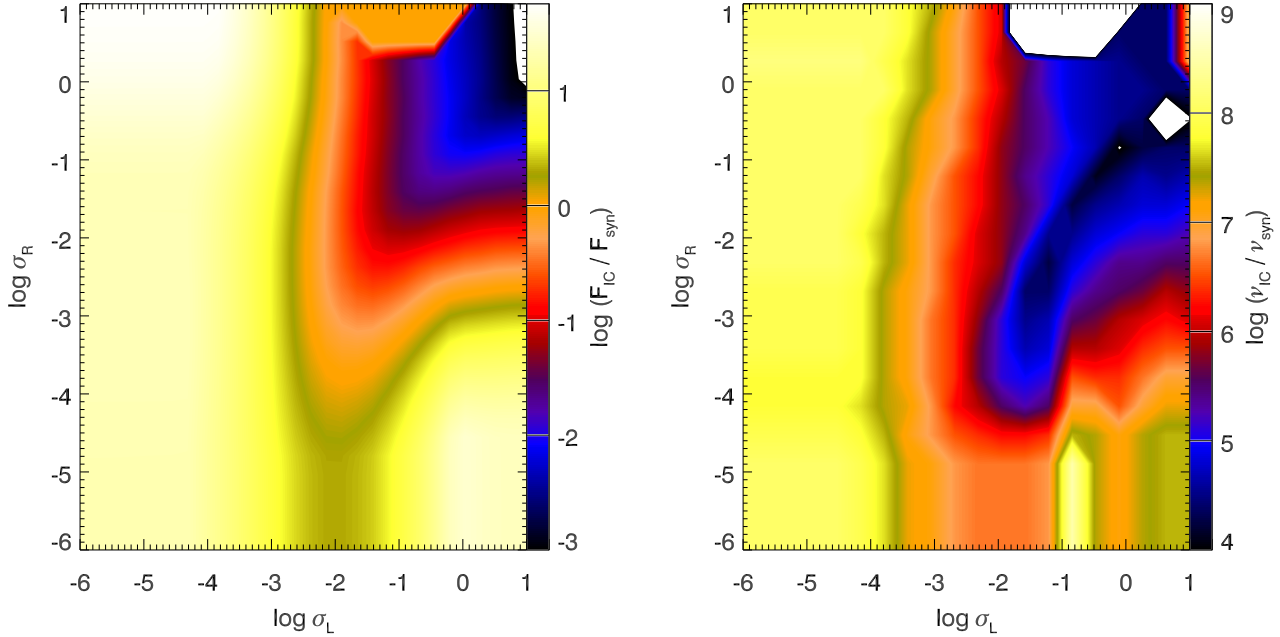


Figure 9. Left panel: contours of the logarithm of the ratio of the IC and synchrotron fluence as a function of the shell magnetization σ_L and σ_R . Right panel: same as left panel, but for the ratio of the frequency of the spectral maxima of the synchrotron and inverse Compton emission.

8.1 Emission mechanisms and magnetization

In Section 6 we show the average spectra and multi-wavelength light curves of three typical models from the parameter space considered in this paper. Synchrotron emission dominates for $\nu \lesssim 10^{17}$ Hz, and is rather independent of the shell magnetization (Fig. 2). The RS dominates synchrotron emission for weakly magnetized shells, while in the case of strongly magnetized shells the FS and RS have comparable contributions. R-band light curves (black lines on Fig. 4) show that the synchrotron emission is due to the slow-

cooling electrons only for the weakly magnetized model, while for shells with $\sigma \gtrsim 0.01$ electrons are fast-cooling.

The SSC emission dominates in the X-ray band and higher frequencies (Fig. 2). However, at early times the synchrotron emission dominates in X-rays (Fig. 5), while in γ -rays the situation is more complex. For the weakly magnetized model (slow-cooling electrons), EIC dominates the early emission, while in the moderate-to-highly magnetized models EIC dominates the late-time emission. The reason for this is that, in the magnetized models, the

high-energy tail of the electron distribution disappears very quickly, so that the incoming synchrotron photons cannot be up-scattered into the 1 GeV range. In the weakly magnetized models there are enough slow-cooling electrons at sufficiently high energies for the SSC to dominate over EIC at later times.

Finally, from Fig. 2 we see that the IC emission is weaker the more magnetized the shells. This is due to the requirement of our model that the shell luminosity (Eq. 1) be constant regardless of σ . From Eq. 3 we see that for $\sigma \gg 1$ the number density in the shells behaves as $\approx \sigma^{-1}$. Since the IC emission depends on the number of electrons (EIC linearly and SSC quadratically), it is clear that the IC emission must necessarily drop for large σ . From the analysis of the three representative models, we conclude that the shell magnetization imprints two main features on the emission properties of blazars. On the one hand, the magnetization changes the ratio of integrated flux below the synchrotron peak to the integrated flux below the IC-dominated part of the spectrum. On the other hand, the magnetization determines whether electrons are slow-cooling (for weakly magnetized shells) or fast-cooling (moderate-to-high magnetization).

8.2 Global trends

We performed a global parameter study (Section 7) to investigate the dependence of some observational quantities on the shell magnetization. As discussed in Section 7, the integrated flux (Fig. 7) follows the trend already shown by the radiative efficiency (Fig. 1). However, the integrated flux and the spectral maxima are quantities dependent on the particular values we have taken in our model, specifically, on the physical size of the shells and their bulk Lorentz factors, as well as on the source redshift. On the other hand, in MA10 we show that the dynamic efficiency is very weakly dependent on the shell bulk Lorentz factor, i.e. it only depends on the shell magnetization for a fixed Δg . In order to eliminate the dependence on absolute quantities in Fig. 9 we show IC-to-synchrotron flux ratio, as well as the ratio of the frequency of IC to the synchrotron spectral maxima. The shape of the contours on the left panel of Fig. 9 does not exactly follow the one in Fig. 1: there is a much stronger dependence on σ_L than on σ_R in most of the scanned parameter space. Nonetheless, in the lower half of the plots, the behavior of both F_{IC}/F_{syn} and f_R is similar. For example, if we keep σ_R constant and equal to 10^{-6} and vary σ_L , we note that the radiative efficiency is larger than 90% for $\sigma_L < 0.01$, then it decays to a local minimum, and successively grows again to reach values in excess of 90% ($\sigma_L \gtrsim 1$). Comparatively, at small values of σ_R the ratio F_{IC}/F_{syn} is close to its maximum for $0.01 \lesssim \sigma_L \lesssim 1$, and touches a minimum in the same interval as f_R . The upper half of Figs. 9 (left) and 1 does not show the same qualitative behavior. The reason for this discrepancy is that RS dominates the emission, and thus the overall radiative properties are more sensitive to the magnetization of the fast shell through which it propagates.

Interestingly, there is a certain degree of degeneration in the values both of the radiative efficiency and of the F_{IC}/F_{syn} ratio considering the regions where one of the two shells is very magnetized and the other is basically non-magnetized (i.e., the upper left and lower right corners of Figs. 9 and 1). In both cases, the radiative efficiency and the fluence ratio are close to their respective maximum values. However, we can distinguish between the case of high- σ_L /low- σ_R and the case of low- σ_L /high- σ_R by looking at the ratio of peak frequencies ν_{IC}/ν_{syn} (right panel of Fig. 9). A noticeably smaller ν_{IC}/ν_{syn} ratio corresponds to the former case than to the later.

The previous analysis suggests that with the combined information of the fluence and peak-frequency ratios, one could try to figure out, by using observational data, which is the rough magnetization of the shells of plasma whose interaction yields flares in blazars. To serve such a purpose, we display in Fig. 10 our models in a 2D parameter space whose horizontal and vertical directions are ν_{IC}/ν_{syn} and F_{IC}/F_{syn} , respectively. We notice that the computed models are distributed in a broad region which, nevertheless, shows a relatively tight correlation between F_{IC}/F_{syn} and ν_{IC}/ν_{syn} . In the left panel of Fig. 10, we display our models in three different colors according to the magnetization of the left shell. The same has been done in the right panel, but for the right shell.

Based on the degree of variation of magnetization between the fast and the slow shells, we have divided the parameter space in three broad regions (labeled with roman numerals **I**, **II** and **III** in Fig. 10), where the shells have the following characteristics:

- I:** moderately magnetized fast shell colliding with a weakly magnetized slow shell, or weakly magnetized fast shell interacting with a strongly magnetized slow one;
- II:** strongly magnetized fast and moderately magnetized slow shells;
- III:** strongly magnetized fast and slow shells.

The first thing to note is that for models in the region **III** both the IC emission and its frequency maximum are lower compared to the rest of the models. This leads us to the conclusion that when the flow is strongly magnetized *and* the magnetization does not vary substantially the IC signature is expected to be relatively weak. Furthermore, region **II** shows that in the case of a larger variation in magnetization (i.e., weakly magnetized slow shell) the frequency maximum remains low, but the IC signature becomes substantially higher. Finally, in the region **I** we see that when the variation in magnetization is more extreme (i.e. a collision of a weakly and a strongly magnetized shells) we get a very strong IC signature and its frequency maximum is shifted to much higher energies.

8.3 Radiative and dynamic efficiency

As discussed in Section 5, the radiative efficiency $\epsilon_e f_{rad}(\epsilon_T + \epsilon_M)$ does not have a one-to-one correspondence to the dynamic efficiency ($\epsilon_T + \epsilon_M$). While the latter peaks in the region $\sigma_L \approx 1$ and $\sigma_R \approx 0.2$, the former reaches its maximum in the region ($\sigma_L \lesssim 10^{-4}$, $\sigma_R \gtrsim 10$). The same can be concluded from the time- and frequency-integrated flux shown in Fig. 7. For purposes of the rest of this discussion we will use f_{rad} as a proxy for the radiative efficiency.

We note that in the region of maximum f_{rad} the FS does not exist. However, we see another region of high f_{rad} in the opposite corner of Fig. 1 the efficiency is quite high as well. Consistent with the discussion in the previous subsection and with what is shown in region **I** in Fig. 10, we conclude that the radiative efficiency is maximal when the variation in magnetization between the colliding shells is large.

8.4 Conclusions and future work

Under the assumption of a constant flow luminosity we find that there is a clear difference between the models where both shells are weakly magnetized ($\sigma \lesssim 10^{-2}$) and those where, at least, one shell has a $\sigma \gtrsim 10^{-2}$. We obtain that the radiative efficiency is largest in those models where, regardless of the ordering, there is a large variation in the magnetization of the interacting shells. Furthermore,

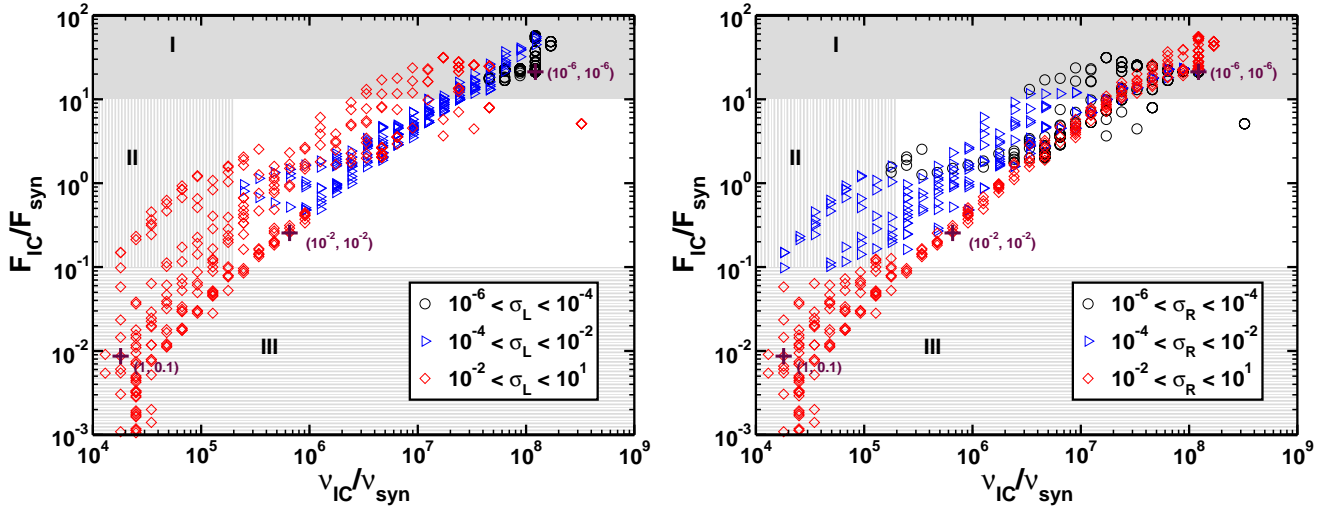


Figure 10. Left panel: F_{IC}/F_{syn} (ratio of the IC to the synchrotron fluence) as a function of ν_{IC}/ν_{syn} (ratio of the maximum spectral frequencies in IC and synchrotron ranges) for the models considered in Sec. 7. The models have been grouped in three bins according to σ_L and are annotated with black circles ($10^{-6} < \sigma_L < 10^{-4}$), blue triangles ($10^{-4} < \sigma_L < 10^{-2}$) and red diamonds ($10^{-2} < \sigma_L < 10^1$). Right panel: same as left panel, but in this case, the models have been grouped according to σ_R in the same bins and same coloring and symbols in as the left panel. Shaded areas denote three regions of interest (see text for details). The three reference models in this paper are marked with purple crosses, and their respective magnetizations are overlaid.

substantial differences between weakly and strongly magnetized shell collisions are observed in the inverse-Compton part of the spectrum, as well as in the optical, X-ray and 1 GeV light curves.

In the previous sections we have deepened our analysis of the radiative efficiency of the process of collision of magnetized relativistic shells of plasma. We have studied this problem from a mostly theoretical point of view, where the intrinsic properties of the flow (in particular the magnetization) have been related to the properties of the resulting (synthetic) spectra and light curves. It is, however, worthwhile to provide suitable links between our theoretical results and the observed properties of blazar flares. Thus, we propose a way to distinguish observationally between weakly magnetized from magnetized internal shocks by comparing the maximum frequency of the inverse-Compton and synchrotron part of the spectrum to the ratio of the inverse-Compton and synchrotron fluence.

For a given flare taken in isolation, our model may predict which is the range of magnetizations, which have to be invoked to fit the spectrum. However, such a model fitting is not fully satisfactory, since it is strongly dependent on the details of the theoretical model. A more generic knowledge of the physical conditions in the flaring regions can be obtained by arranging the observational data in plots where the fluence ratio F_{IC}/F_{syn} is represented versus ν_{IC}/ν_{syn} . The reason being that the fluence and frequency ratios are redshift and *source* independent, since they are mostly influenced by the variation of the bulk magnetization of the blazar jets (assuming that the viewing angle is fixed). We note that different flares of the same blazar, as well as different flares of distinct blazars can be plotted in such graphs and compared with our theoretical predictions. In addition, an average over a number of flares of the same source might also be interpreted using our model if the magnetization ratio of different pairs of colliding shells is similar.

Our results suggest that the variability in the flow magnetization is a factor that shall be considered to explain the observed continuity of properties of the blazar sequence (e.g., Fossati et al. 1998; Ghisellini et al. 1998; Ghisellini & Tavecchio 2008). Looking at Figs. 2 and 3, it is evident that if the magnetization of the

shells is not too large ($\sigma_{L,R} \lesssim 10^{-2}$), increasing the flow magnetization shifts the synchrotron peak towards the UV band and lowers the IC peak. Leaving aside orientation effects (which we are neglecting here since we fix the viewing angle), such a behavior suggests that LBL blazars may correspond to barely magnetized flows, while HBL blazars could correspond to moderately magnetized ones. If the magnetization is large ($\sigma_{L,R} \gtrsim 0.1$) the synchrotron peak shifts towards lower frequencies, and the IC spectral peak falls three orders of magnitude below the synchrotron peak. The latter situation seems not to be observed and, thus, we conclude that this is an indication that the typical value of the magnetization in the flow of blazars is $\sigma \lesssim 10^{-2}$. We note that this value is about one order of magnitude smaller than that suggested for the flow in gamma-ray bursts (e.g., Giannios & Spruit 2006).

Our results are not in contradiction with the common view, according to which, the variation of the properties of the blazar family correspond to the changes in the bolometric luminosity of the synchrotron component ($L_{bol,sync}$; Fossati et al. 1997). In such a case LBLs and HBLs are extrema of a one-parameter family, where LBLs are more radio luminous than HBLs. However, $L_{bol,sync}$ is not a “direct” physical property of the plasma in a relativistic jet, since the same $L_{bol,sync}$ may arise with an infinite number of combinations of bulk Lorentz factors, blazar orientations with respect to the line of sight, flow magnetization, etc. In this paper, we explore the role that the flow magnetization (a direct physical property of the emitting plasma) plays in shaping not only $L_{bol,sync}$, but also the whole spectrum. We conclude that the flow magnetization alone is enough to explain the difference in $L_{bol,sync}$ and in the high-energy part of the spectrum (i.e., in the Compton-dominated regime) found in the blazar family. We also point out that Fossati et al. (1998) also arrive to the same qualitative conclusion, since they explain that fixing the bulk Lorentz factor, and assuming that the SSC model be valid for all sources, the spectral differences in the blazar sequence shall result from a systematic variation in magnetic field strength, HBLs having the highest random field intensity. We go in this paper a step further. When considering the dynamical changes induced by non-negligible *macroscopic* magnetic fields, the same conclusion

as in Fossati et al. (1998) holds, but we remark that, in our case, the total jet luminosity is kept constant (by construction of our models), only the magnetization is varied.

A problem with the internal shock scenario is that it is apparently not able to explain the ultra-fast variability of TeV blazars (e.g., Albert et al. 2007; Aharonian et al. 2007). In order to properly account for this fast variability (on the time scales of minutes) fast “minijets” (Giannios et al. 2009) or “spines/needles” (Tavecchio & Ghisellini 2008) need to exist in a much larger, slower jet. We do not consider minijets in our current models, although they will need to be considered in the future.

In the future work we will improve our modeling by including the resistive dissipation as a source of energy for the radiating non-thermal particles. This will make it possible to assess whether radiative and dynamical efficiencies have a one-to-one correspondence or if there is a fundamental degree of independence. In the latter case it might be difficult to infer the flow properties except in those asymptotic cases where the dissipation either does not play a significant role or it dominates the dynamics. Furthermore, we will study how changing the viewing angle and the Doppler factor reflect on the observational signature. Finally, we will include the effects of the presence of a thermal component during particle injection at very magnetized shocks (Sironi & Spitkovsky 2009; Giannios & Spitkovsky 2009).

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APPENDIX A: COMPARISON OF THE DYNAMIC EFFICIENCY DEFINITIONS

In this appendix we compare two alternative definitions of the dynamic efficiency, one due to MA10 and the other inspired by Narayan et al. (2011, NKT11 in the following).

MA10 define in their Eq. 12 three components (kinetic, thermal and magnetic, respectively) of the total energy in each shell:

$$\begin{aligned} E_K(\Gamma, \rho, \Delta x) &:= \Gamma(\Gamma - 1)\rho c^2 \Delta x \\ E_T(\Gamma, \rho, p, \Delta x) &:= [(\rho \varepsilon + p)\Gamma^2 - p]\Delta x, \\ E_M(\Gamma, \rho, \sigma, \Delta x) &:= \left(\Gamma^2 - \frac{1}{2}\right)\rho \sigma c^2 \Delta x \end{aligned} \quad (\text{A1})$$

where ρ , p , ε , σ and Γ are the fluid rest-mass density, thermal pressure, specific internal energy density, magnetization parameter and the Lorentz factor. Δx is the width of the shell. Then, the total dynamical efficiency is defined as the ratio of the sum of the magnetic and thermal energies in the post-shock state to the total initial energy (MA10).

Recently, NKT11 have computed the radiative efficiency of

magnetized internal shocks using a slightly different definition. The definition of NKT11 has the advantage of resulting into a radiative efficiency, which is Lorentz-invariant, while the one of MA10 is not. Therefore, the dynamic efficiency can be computed in the lab frame and then used in the rest frame of the contact discontinuity to be directly compared with the radiative efficiency, as was done in Sec. 5. However, NKT11 approach does not shed any light on the problem of obtaining a true energetic efficiency in terms of the initial conditions of the shells. This is because their definition does not consider the efficiency of conversion of the *initial* total energy of the shells to radiation but, instead, the efficiency of conversion of the enthalpy per particle *after* the shell collision into radiation enthalpy.

With the aim of introducing a Lorentz invariant, energy-based definition of the radiative efficiency in terms of a quantity akin to the dynamic efficiency of MA10, we consider the following expressions for the kinetic, thermal and magnetic energies

$$\begin{aligned} \hat{E}_K(\Gamma, \rho, \Delta x) &:= \Gamma^2 \rho c^2 \Delta x \\ \hat{E}_T(\Gamma, \rho, p, \Delta x) &:= \Gamma^2 (\rho \varepsilon + p) \Delta x, \\ \hat{E}_M(\Gamma, \rho, \sigma, \Delta x) &:= \Gamma^2 \rho \sigma c^2 \Delta x \end{aligned} \quad (\text{A2})$$

With these definitions, the ratio of energies shown in Eq. 31 becomes Lorentz invariant, and so is the total dynamic efficiency. Furthermore, when calculated in the LAB-frame, the exact values of the dynamical efficiency computed using any of the two sets of definitions (Eqs. A1 or A2) differs very little, as it is shown in Fig. A1.

APPENDIX B: COMPUTATIONAL REQUIREMENTS FOR REALISTIC CALCULATIONS

In this paper we have adopted a cylindrical geometry and ignored adiabatic losses of the non-thermal electrons, as well as the high-latitude emission. However, we have included both of these effects in previous works studying afterglows of gamma-ray bursts (Mimica et al. 2010; Mimica & Giannios 2011). In those calculations, performed using the radiative transfer code *SPEV* (Mimica et al. 2009), we have only taken into account the synchrotron and EIC emission processes. Even so, the calculation of a realistic multi-wavelength light curve of a *single* model lasts anywhere between few hours to few days on a supercomputing cluster with several hundreds of computing cores, depending on the number of wavelengths at which the emission is computed. If the synchrotron-self absorption (SSA) is included, then the parallel scalability of the method is reduced and the calculations can become prohibitively expensive (see footnote 3 of Mimica et al. 2010).

In this paper we have a similar problem due to the fact that we include the SSC process, which, as SSA, is non-local and difficult to parallelize in a realistic conical geometry. The use of cylindrical geometry and other simplifying assumptions has enabled us to nevertheless compute light curves of 900 models for 96 frequencies at 120 observer times. This required 200 thousand computing hours on the *MareNostrum* computer of the Barcelona Supercomputing Center. Based on our previous experience, we estimate an order of magnitude more resources are needed if the adiabatic cooling of the electrons is included, and another order of magnitude if an accurate radiative transfer method such as *SPEV* is used instead of approximate method described in Section 4. We note that such a

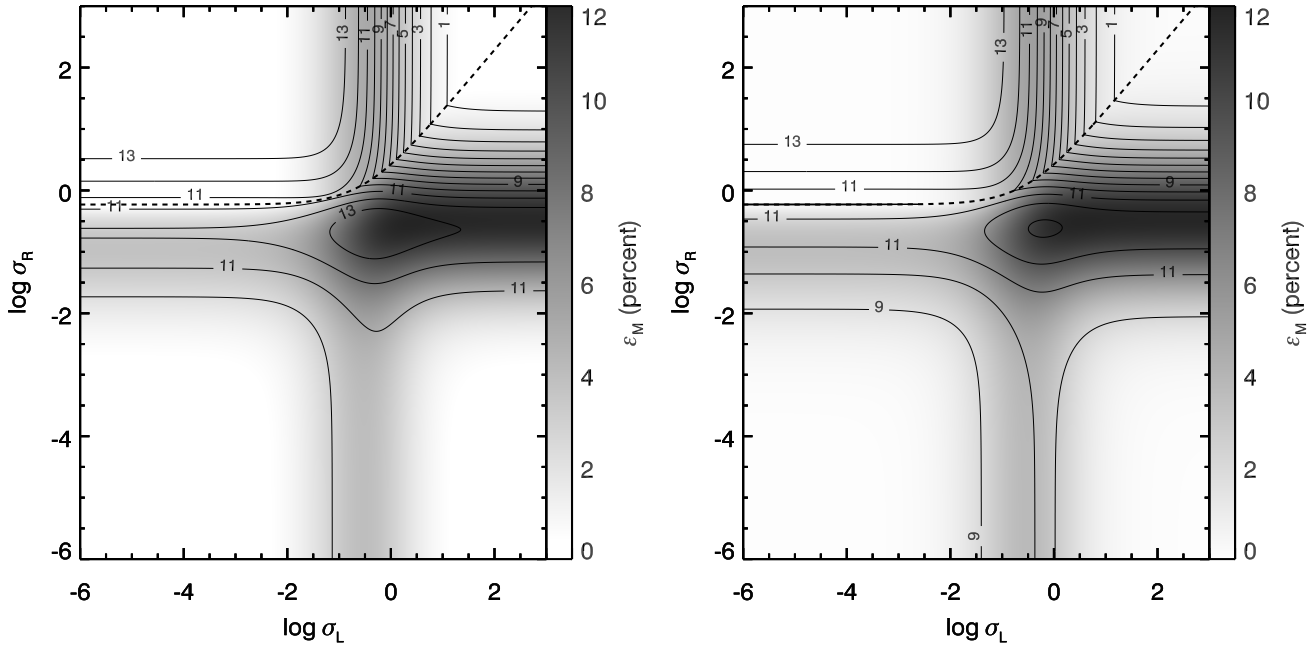


Figure A1. Left panel: contours of the total dynamic efficiency for the model parameters from Table 1 computed using definition of MA10 (see Eq. A1). Contours indicate the efficiency in percent and their levels are 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and 13. In the region of the parameter space above the dashed line there is no forward shock. Filled contours show the magnetic efficiency. Right panel: same as left panel, but using Eq. A2, inspired by NKT11.

calculation requires resources in the range of 10 million computing hours on a computer such as *MareNostrum*.

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